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Particulate flow with sublimation or evaporation and with thermal and hydrodynamic non-equilibrium

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Abstract—A model is developed for the description of the process of particle sublimation or droplet evaporation in a gaseous environment. The conservation and closure equations for the process are derived and are cast in dimensionless form. The system of equations is solved numerically for the case when the mixture of the two phases is subjected to a step increase in wall temperature. The effect of several parameters (such as particle sizes, physical properties and size of the wall temperature step) is examined on the rate of evaporation of the condensed phase. The temperatures and velocities of the two phases during the phase change are also calculated and reported.

INTRODUCTION

THE SUBJECT of heat transfer in suspensions was first examined in the 1960s, when suspensions were considered as an alternative to high pressure gases for nuclear reactor cooling. The experimental studies of Farbar and his co-workers [1, 2], Tien and Quan [3] and Danziger [4], among others, provided the engineering correlations for the convective film coefficient of a suspension. A review of most of the experimental data and the resulting correlations can be found in Pfeffer *et al.* [5]. Regarding the analytical studies on the subject, Tien [6] made the first one for homogeneous suspensions of very low solids content. Ozbelge and Sommer [7] and Michaelides [8] later developed different analytical and computational models for the heat transfer of suspensions.

In the case of particulate flows experimental data on temperatures of particles and evaporation (or sublimation) rates are non-existent, because of the difficulty of the measurements. Analyses of homogeneous suspension flows, which preclude thermal and hydrodynamic non-equilibrium effects, predominate among the analytical approaches to the subject. Regarding non-equilibrium flows of droplets and particles a study by Whalley *et al.* [9] treats the subject of droplet flow in a gaseous stream of the same material. The flow of particles with thermal and hydrodynamic non-equilibrium, when there is a temperature step at the wall or when hot particles are injected in a developed flow is examined in ref. [10].

The present study develops a model for the flow and heat transfer of evaporating droplets or sublimating particles under conditions of thermal and hydrodynamic non-equilibrium in a duct. The behavior of rigid particles in a gaseous stream (of the same or different material) is studied, when the velocities and temperatures of the two phases are different. In this case there is momentum, heat and mass transfer between the two phases, which results in the acceleration of the mixture. The emphasis of the study is on the rate of phase change of the condensed phase as well as on the mechanical and thermal non-equilibrium that exists between the two phases, which manifests itself in the difference between the velocities and the temperatures of the two phases. The rate of sublimation (or evaporation) of the dispersed phase is calculated and from it the instantaneous mass and radii of the particles. Phase velocities and temperatures are also computed.

THE SET OF EQUATIONS FOR A SUSPENSION

The flow of sublimating particles in a duct is assumed to be one-dimensional with the model of particles bouncing on the walls as described in ref. [11]. For the simplicity of the calculations all particles are assumed to have the same size; however, the theory covers the polydisperse suspensions and the model can be adjusted to take them into account with minor modifications. The diameter of the particles in all cases considered is small ($d/D \ll 1$). The gas velocities and the Reynolds numbers considered are such that the flow is turbulent and the particles are in suspension. The particle concentrations by volume are always

NOMENCLATURE

C	concentration
C_D	drag coefficient
c	specific heat
D	diameter
d	diameter of a particle
F_D	drag force
f	friction coefficient
G	mass flux
g	gravity
h	heat transfer coefficient
h_{fg}	latent heat
k	thermal conductivity
m	mass
N	number of droplets
N_M, N'_M	dimensionless heat ratios
Nu	Nusselt number
P	pressure
Pr	Prandtl number
Q	heat transfer
r	particle radius
Re	Reynolds number
T	temperature
t	time
U	velocity
v_{fg}	volume change upon phase transformation.

Greek symbols

δ	specific heat ratio
μ	viscosity
ν	cinematic viscosity of the gas
ρ	density
σ	Stefan–Boltzmann constant
τ	characteristic time of the particles.

Subscripts

D	pipe diameter
g	gas
p	particles
w	wall
0	initial.

Superscripts

*	dimensionless quantities.

small (<0.01), thus ensuring negligible particle interactions. Under these conditions the conservation equations for the motion of gas and particles are as in the following sections.

Continuity equation

A combined equation for the conservation of mass of the particles and of the gas may be written as follows:

$$NU_p(\tfrac{4}{3}\pi\rho_p r^3) + U_g\rho_g = G_0 = \text{const.} \tag{1}$$

where N is the number density of the particles (particles per m^3), which remains constant, U the longitudinal velocity, ρ the density, r the radius of the particles (variable because of the change of phase) and G_0 the total mass flux. Subscripts p and g refer to the particles and the gas, respectively. In this study the density of the particles is assumed to be constant; the density of the gaseous phase is given as a function of the pressure and the temperature. The continuity equation may be written in differential form as follows:

$$N\tfrac{4}{3}\pi\rho_p\left(r^3\frac{dU_p}{dt} + 3U_p r^2\frac{dr}{dt}\right) + \rho_g\frac{dU_g}{dt} + \frac{d\rho_g}{dt}U_g = 0. \tag{2}$$

This equation shows that evaporation or sublimation results in an acceleration of the gaseous phase (and subsequently of the particles themselves) because of the creation of the excess mass of the gas.

The momentum equations for the two phases

The force balance on a particle of radius r may be written as follows in its complete form:

$$\tfrac{4}{3}\pi\rho_p\frac{d}{dt}(r^3U_p) = F_D - \tfrac{4}{3}\pi r^3\frac{dP}{dx} + \tfrac{2}{3}\pi r^3\rho_g\left(\frac{dU_g}{dt} - \frac{dU_p}{dt}\right) + 6(\pi v)^{0.5}r^2\rho_g\int_0^t \frac{\frac{dU_g}{dt} - \frac{dU_p}{dt}}{(t-t')^{0.5}}\,dt' + \tfrac{4}{3}\pi r^3 g(\rho_p - \rho_g) \tag{3}$$

where g is the acceleration due to gravity and v the dynamic viscosity of the gas.

The left-hand side of the above equation represents the instantaneous change of momentum of the particle. The first term on the right-hand side represents the drag force. The second term is due to the pressure gradient; this term is negligible, unless the particles are in the presence of very strong pressure gradients, a case very seldom met. The third term is due to the added mass of the particles because of the acceleration of a small quantity of gas with it. This term is of importance when the density of the gas is comparable to or higher than that of the particles. In the cases examined here $\rho_p/\rho_g \gg 1$ and the added mass term is neglected also. The fourth term represents the so-called Basset force, which is due to the history of the acceleration of the particles. The Basset force may become important when the particles are very small; the effects of this force are examined in this study, as reported in the section of the results. Finally the last term represents the action of the gravity forces on the particles.

The viscous drag force F_D is given by an empirical relation in terms of the relative velocity of the particles

$$F_D = \tfrac{1}{2}C_D\pi r^2\rho_g|U_g-U_p|(U_g-U_p) \qquad (4)$$

where C_D is the drag coefficient, a function of the Reynolds number of the particles. Since the latter in this study can be appreciably greater than 1 we chose the following expression from refs. [12, 13]:

$$C_D = \frac{24}{Re_p}(1+0.15Re_p^{0.687}) \qquad (5)$$

where the Reynolds number of the particles is defined as usual in terms of the relative velocity of the particles and the particle diameter.

The momentum equation for the gas is used in the form of the pressure drop equation as follows:

$$\frac{dP}{dx}+\frac{1}{2D}f\rho_g U_g^2+\frac{d}{dx}(\rho_g U_g^2) = 0 \qquad (6)$$

where D is the diameter of the pipe, x the longitudinal distance and f the friction factor for the suspension. The latter is different from the friction factor of the gas flowing alone, because of the interactions of particles with gas and because of bouncing on the walls of the duct. The following expression for the friction factor f is adopted, emanating from the study of particle flows in ref. [11]:

$$f = f_0+0.072m^*\frac{(Dg)^{0.5}}{U_g f_0} \qquad (7a)$$

where m^* is the loading of particles (ratio of solids to gas mass fluxes) and f_0 the friction factor for the gaseous phase flowing alone in the duct, here given by the following closure equation:

$$f_0 = 4(0.0014+Re_g^{-0.32}). \qquad (7b)$$

The heat transfer and energy equations

Evaporation of the particles occurs because of the heat transfer from the gas to the particles or from the hotter walls of the duct to the particles. Convection is the predominant mode of heat transfer from the gas to the particles and from the wall to the gas, while radiation is the predominant mode for the heat transfer from the wall to the particles. Conduction from the walls to the particles is neglected for the following two reasons. First, the time of collision of the particles with the wall is very small for appreciable amounts of energy to be conducted to the particles. Second, the particle concentrations (by volume) are very small; therefore at any moment there are very few particles in the immediate vicinity of the wall, where any thermal interactions by conduction have to take place. For simplicity the gaseous phase is considered transparent and the particles completely opaque. However, the model can be easily modified to take into account the effects of a gray gas or partly reflecting particles.

The velocities of the two-phase mixture are low enough, so that critical conditions are not met in the flow domain. Therefore, there is no reason to expect thermodynamic non-equilibrium between the gaseous phase and the particles. This is equivalent to saying that frozen flow is not expected in the applications considered for this study. In addition the density ratio of the condensed to the gaseous phase is very large, for the Kelvin–Helmholtz effect [13] to be negligible. Hence, the solids temperature is a function of the partial pressure of its vapor in the gaseous phase. If the gaseous phase is composed of one component (the same as that of the sublimating particles), then the particle temperature is given as a function of the total pressure; therefore, changes of the pressure result in particle temperature changes according to the Clausius–Clapeyron relation. The rate of change of temperature is then written as follows:

$$\frac{dT_p}{dt} = \frac{Tv_{fg}}{h_{fg}}\left(\frac{dP}{dt}\right) \qquad (8)$$

where h_{fg} is the latent heat of the substance, v_{fg} the volume change upon phase change and T the absolute temperature. In the case where the gas is composed of more than one substance P should be substituted by the partial pressure of the vapor of the condensed phase.

The heat absorbed by the particles is through convection from the gas and radiation from the walls as explained above. The rate of heat flux absorbed by the particles is written as follows:

$$\dot{Q}_p = 4\pi r^2 N h_p(T_g-T_p)+4\pi r^2\sigma N(T_w^4-T_p^4) \qquad (9)$$

where σ is the Stefan–Boltzmann constant and h_p the convective heat transfer coefficient for the particles, which must be supplied by an empirical relation. The problem of inter-particle shadowing is of no importance in the applications considered here because of the low concentrations of the particles.

The heat flux absorbed by the particles manifests itself in two ways: it causes the evaporation of some of the mass of the particles or it contributes to the change of the temperature of the substance. Thus, one may write the following expression for the rate of mass per unit volume undergoing phase change:

$$\dot{m}_s = \frac{\dot{Q}_p - Nm_p c_p\left(\frac{dT_p}{dt}\right)}{h_{fg}} \qquad (10)$$

where h_{fg} is the latent heat of the substance (for sublimation or vaporization) and c_p the specific heat of the particles. Given that the rate of temperature change of the particles is controlled by the pressure change, as in equation (8), the last two equations yield the rate of mass exchange (per unit volume) between the condensed and the vapor phase. The latter is related to the rate of change of the radius of the particles by the mass balance equation

$$\dot{m}_s = -\frac{\mathrm{d}}{\mathrm{d}t}(Nm_p) = -4\pi r^2 N\rho_p \frac{\mathrm{d}r}{\mathrm{d}t}. \quad (11)$$

Heat flux is also transferred to the gaseous phase from the wall at a rate, which is given by the following relation:

$$\dot{Q}_g = \frac{4h_w}{D}(T_w - T_g) \quad (12a)$$

where h_w is the convective film coefficient, which must be given by an empirical relation and D the pipe diameter. With the heat fluxes to the particles also known, the energy equation for the gaseous phase is reduced to an expression for the temperature change of the gas

$$\rho_g c_{pg}\left(\frac{\mathrm{d}T_g}{\mathrm{d}t}\right) = \frac{4h_w}{D}(T_w - T_g) + 4\pi r^2 N h_p (T_p - T_g) \quad (12b)$$

where c_{pg} is the specific heat of the gas at constant pressure. In the above equations the particles are assumed to absorb the radiation completely. The effects of finite reflectivity can be easily incorporated in the heat flux equations by including the appropriate reflectivity and absorbtivity coefficients.

Two closure equations are needed for the two heat transfer coefficients h_p and h_w. In this study the expressions advocated in refs. [4, 5, 10, 12] have been used

$$h_p = \frac{k}{2r}(2 + 0.6 Re_p^{0.5}\ Pr^{0.33}) \quad (13a)$$

and

$$h_w = 0.023\frac{k}{D} Re_g^{0.8}\ Pr^{0.4}(1 + 4 Re_g^{-0.32}\ m^*\delta). \quad (13b)$$

In the above equations k is the gas conductivity, Pr the Prandtl number for the gas, Re_g the Reynolds number based on the gas velocity and the pipe diameter and δ the ratio of the specific heat of the particles to that of the gas ($\delta = c_p/c_{pg}$). The mass flux ratio m^* (loading of particles) is given in terms of the flow variables by the following expression:

$$m^* = \frac{Nm_p U_p}{\rho_g U_g} = \frac{4/3\pi r^2 \rho_p N U_p}{\rho_g U_g}. \quad (14)$$

DIMENSIONLESS FORM OF THE EQUATIONS

The conservation and closure equations of the above section constitute a system of simultaneous ordinary differential equations. For the solution of the system it is necessary to know the initial conditions of the two flowing phases. It is desirable to transform this initial value problem to a dimensionless set of equations. There is a natural time scale to the problem, namely the characteristic time of the particles

$$\tau = 4r^2\rho_p/18\mu \quad (15)$$

which appears in the momentum and energy equations of the particles. Since r is a variable in this problem and, therefore, cannot be used for making the equations dimensionless it was decided to use the value of τ at time $t = 0$ ($\tau_0 = 4r_0^2\rho_p/18\mu$) for obtaining the dimensionless time in all the equations. The initial values of gas velocity, U_{g0}, and particle radius, r_0, were used for the transformation of the other flow variables. The wall absolute temperature was used for making the temperatures dimensionless. Thus, the following dimensionless variables have been chosen for the transformation of the equations

$$t^* = \frac{18t\mu}{4\rho_p r_0^2} \quad (16a)$$

$$r^* = r/r_0 \quad (16b)$$

$$U_g^* = U_g/U_{g0}, \quad U_p^* = U_p/U_{g0} \quad (16c)$$

$$T_g^* = T_g/T_w, \quad T_p^* = T_p/T_w. \quad (16d)$$

With the choice of the above dimensionless quantities it is advantageous to make the density dimensionless also by using the gas density which corresponds to the wall temperature, ρ_{gw}, and the rate of change of phase by using the initial characteristic time τ_0 of the particles

$$\rho_g^* = \rho_g/\rho_{gw} \quad (16e)$$

and

$$\dot{m}_s^* = \frac{\dot{m}_s}{18\pi\mu r_0 N}. \quad (16f)$$

Under these conditions the working equations are transformed as follows.

Mass conservation of all the species

$$C_0\left(r^{*3}\frac{\mathrm{d}U_g^*}{\mathrm{d}t^*} + 3U_p^* r^{*2}\frac{\mathrm{d}r^*}{\mathrm{d}t^*}\right) + \frac{\rho_{gw}}{\rho_p}\left(\rho_g^*\frac{\mathrm{d}U_g^*}{\mathrm{d}t^*} + U_g^*\frac{\mathrm{d}\rho_g^*}{\mathrm{d}t^*}\right) = 0 \quad (17)$$

where C_0 is the initial concentration of the particles, equal to $4N\pi r_0^3/3$.

Particle momentum equation

$$\frac{\mathrm{d}U_p^*}{\mathrm{d}t^*} = \frac{1}{r^{*2}}(U_g^* - U_p^*)(1 + 0.15 Re_p^{0.687}). \quad (18)$$

Energy equation for the gas

$$\rho_g^*\frac{\mathrm{d}T_g^*}{\mathrm{d}t^*} = \frac{8}{9Pr}\left(\frac{\rho_p}{\rho_{gw}}\right)\left[Nu_D(1 - T_g^*)\left(\frac{r_0^2}{D^2}\right) + \tfrac{3}{4}C_0 r^{*2} Nu_p (T_p^* - T_g^*)\right] \quad (19)$$

where the Nusselt number for the particles Nu_p and the Nusselt number for the pipe Nu_D emanate from a dimensionless form of equations (13a) and (13b).

Rate of phase change of the particles

$$\dot{m}_s = -r^{*2}\frac{\mathrm{d}r^*}{\mathrm{d}t^*} = \tfrac{2}{9}N_M\left[\frac{Nu_p}{\delta Pr}(T_g^* - T_p^*) - \tfrac{3}{2}r^*\left(\frac{\mathrm{d}T_p^*}{\mathrm{d}t^*}\right)\right]r^{*2} + \tfrac{2}{9}N'_M r^{*2}(1 - T_p^{*4}). \quad (20)$$

The dimensionless parameter N_M represents a ratio of sensible to latent heat and is equal to

$$N_M = \frac{c_p T_w}{h_{fg}} \quad (21a)$$

while the parameter N'_M represents the ratio of radiative to latent heat and is equal to

$$N'_M = \frac{\sigma T_w^4 r_0}{\mu h_{fg}}. \quad (21b)$$

The product δPr (which is equal to $c_p\mu/k$) may be interpreted as a Prandtl number based on the specific heat of the particles.

Equations (17)–(20) represent the dimensionless form of the system of ordinary differential equations for the variables U_g^*, U_p^*, T_g^* and r^*. The other dimensionless flow variables, such as T_p^* and ρ_g^* are obtained from closure equations for the properties of the species and the conditions for thermodynamic equilibrium of the two phases, such as the dimensionless form of equation (8). With the initial conditions for the velocities and temperatures of the phases and the initial radius and number of particles, the above equations form a well posed non-linear system, which may be solved by a numerical method.

PARTICULATE FLOW PAST A WALL TEMPERATURE STEP

As an application of the subject of heat transfer in suspension flows with phase change we consider the case where the wall temperature of the duct undergoes a temperature step from T_{w0} to T_w at time $t = 0$. For simplicity it is assumed that at $t = 0$ the solids and gas are at thermal and hydrodynamic equilibrium, that is the particles and gas velocities and temperatures are equal. This equilibrium state arises in the fully developed flow of a two-phase mixture with small particles. Because the effects of radiation are taken into account there is no restriction on the magnitude of the temperature step imposed, as for example in ref. [10].

When the suspension enters the domain of higher temperature T_w the gas is heated up and accelerates, thus causing the acceleration of the particles. At the same time heat is convected to the particles from the gas and radiated from the walls of the duct, causing some of the particles to sublimate. The phase change is a source of mass for the gaseous phase resulting in further acceleration of the gas and of the whole mixture subsequently. The process of phase change and acceleration stops when all the condensed phase has evaporated and the resulting gas is heated up to the wall temperature. It must be pointed out that the acceleration of the gas causes a relative velocity between the gas phase and the particles; thus, the Reynolds number of the particles becomes finite, a phenomenon which results in higher acceleration and convective heat flux from the gas to the particles.

RESULTS

Numerical calculations were made for the temperature step case based on the model described above. The working equations were solved by an explicit time-marching scheme. For accuracy and for computational stability the time step in this scheme was taken as a fraction of the characteristic time of the particles τ. The results of the computations are shown in the figures that follow.

Figure 1 shows the evolution over time of the dimensionless radius of water droplets as they undergo a temperature step in the pipe from an initial value of $T_p = 120$°C to a value of T_w, which is shown on the graph as a parameter (500, 700 and 1000°C). The value of C_0 for all the curves is 0.00419. This number is obtained by having 10^9 droplets of water of initial radius 0.1 mm in the flow field; the initial mass loading corresponding to this concentration is 3.74. The common characteristics of all three parametric curves are that initially the radius decreases at a slower rate and finally the rate of decrease becomes very fast. This is due to the fact that the mass of a droplet is proportional to the cube of its radius and, therefore, when the droplets become very small a low amount of evaporated mass results in a large radius drop. The droplets which experience higher wall temperatures evaporate faster, as expected.

Figure 2 depicts the dimensionless temperatures of the vapor phase vs the dimensionless time t^* for the same parameters as Fig. 1. Because temperatures are made dimensionless by dividing by the wall tem-

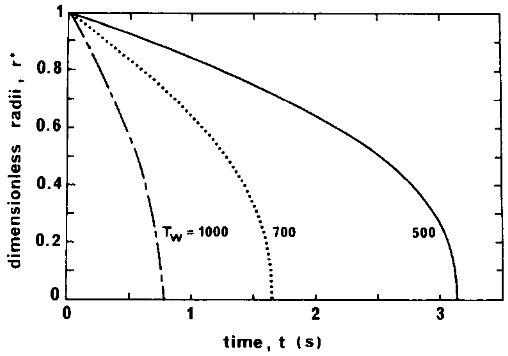

FIG. 1. Water droplet evaporation for wall temperatures of 500, 700 and 1000°C with $C_0 = 0.00419$ and $T_p = 120$°C.

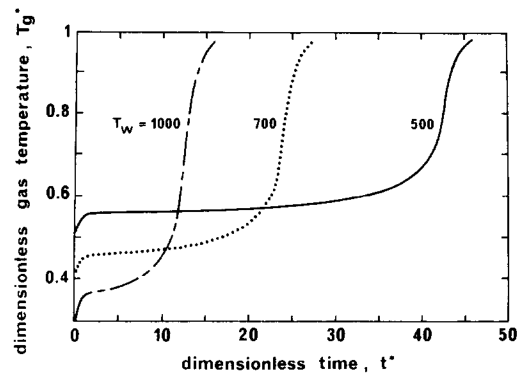

FIG. 2. Temperatures of the gaseous phase vs dimensionless time with $C_0 = 0.00419$ and $T_p = 120°C$.

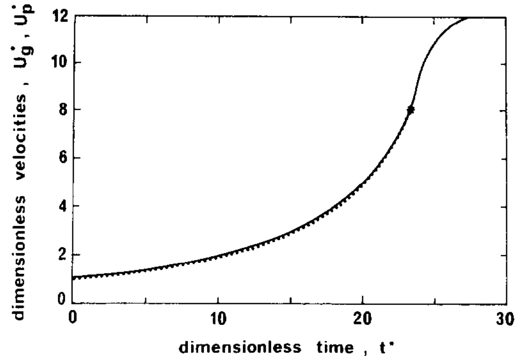

FIG. 3. Dimensionless velocities of the two phases for $T_w = 700°C$ with $C_0 = 0.00419$ and $T_p = 120°C$.

perature, the starting points in the three cases are different. The curves show a fast increase in the gas temperatures initially, followed by a very slow rate of increase, and then by a second faster rate, until the value 1 ($T_g = T_w$) is reached asymptotically. The first fast rate is due to the fact that a great deal of heat is absorbed from the wall by the gas and not much from the gas by the particles (this, because initially gas and particles are at the same temperature and heat transfer between the two phases is low). When the gas reaches a higher temperature between T_{g0} and T_w, the convection of heat from the gas to the particles becomes dominant in the heat balance and most of the heat flux from the wall to the two-phase mixture is absorbed by the droplets. Therefore, the temperature of the vapor phase is almost constant as may be seen in the figure. When the droplets become very small (and hence the convective heat transfer to them becomes very low) the rate of increase of the gas temperature is faster. High heat transfer is also observed when the droplets have evaporated (heat transfer takes place according to the single-phase equations) in which case all the heat flux is absorbed by the gas. The dimensionless temperature then approaches asymptotically its equilibrium value 1. It must be pointed out here that the calculations showed that the temperature of the droplets decreases by a small amount, following the pressure, because of the imposed thermodynamic equilibrium condition.

The changes in the vapor temperatures are also reflected in the velocities of the two phases. The dimensionless velocities of the vapor and of water droplets are shown in Fig. 3. This figure is again drawn with the initial parameters being the same as in the previous two figures. The temperature at the wall is 700°C. The initial slow velocity increase of the vapor phase is due mainly to the slow rise of the vapor phase temperature and the evaporation of the droplets. Afterwards, there is a rather fast acceleration of the gas velocity because its density decreases fast (following the temperature). From this point onwards all the increase of the velocity of the vapor is due to the temperature increase of the vapor phase. The velocity of the droplets, simply follows the vapor velocity (and lags by a small amount as seen in the figure). When the radius of the droplets becomes very small the droplet velocity is very close to that of the vapor (in the figure they almost touch). At this point all evaporation has taken place and afterwards there is only single-phase flow. This point is shown by an asterisk on the figure.

The influence of the Basset forces on the acceleration of the particles, which is touted to be of importance when particle sizes are very small, was also determined. The computer algorithm was modified to take into account the Basset forces and several runs were made with and without the Basset forces for comparison. Differences on the instantaneous acceleration of the particles as high as 20% were observed by the inclusion of the Basset forces. However, the effect of the Basset forces on the integral quantities were very small in all the cases examined. For example it was found that in all cases the effect of the inclusion of Basset forces on the time of droplet evaporation is less than 3%, and the effect on the final velocity of the particles and gas less than 1%.

Figure 4 shows for the same initial conditions and parameters the loading for the three wall temperatures of 500, 700 and 1000°C. The loading can be inter-

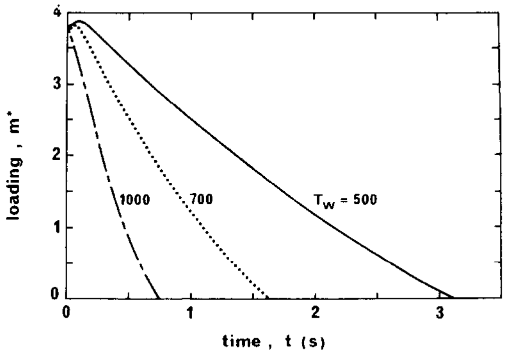

FIG. 4. The evolution of loading in time for wall temperatures of 500, 700 and 1000°C with $C_0 = 0.00419$ and $T_p = 120°C$.

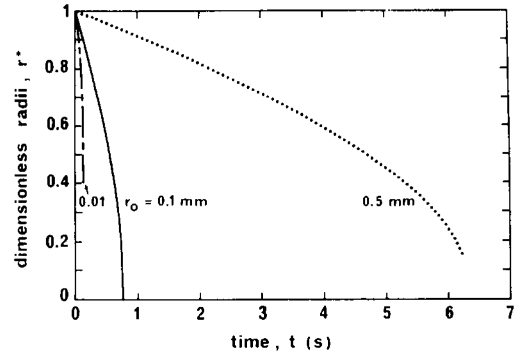

FIG. 5. The effect of the initial droplet radius on the rate of evaporation for $T_w = 1000°C$ with $C_0 = 0.00419$ and $T_p = 120°C$.

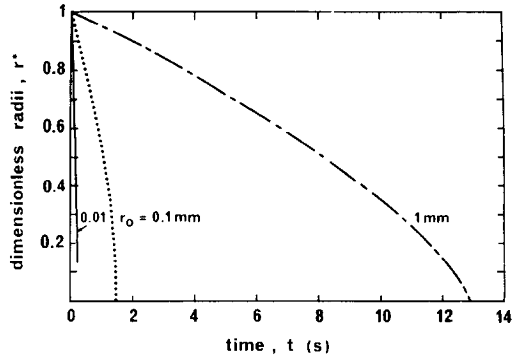

FIG. 6. The effect of the initial droplet radius on the rate of evaporation for mercury with $C_0 = 0.00419$ and $T_p = 120°C$.

preted as the amount of liquid by mass, which remains in the two-phase mixture. It is observed that the loading decreases very rapidly. Even at the lowest wall temperature of 500°C less than 5% of the liquid remains after 3 s.

One parameter, which influences the results of the evaporation considerably is the initial radius r_0. In the dimensionless representation of the equations it appears in the concentration parameter C_0 and as the ratio r_0/D in the heat transfer equation. Its influence lies in the fact that the heat flux to the particles is proportional to the square of the radius, while the concentration is proportional to the cube of the radius (for this reason the ratio r_0/D appears only in the heat transfer equation). The effect of the initial radius on the evolution of the droplet size is shown in Fig. 5; the substance is again water and the concentration for all three curves is 0.00419. T_w is 1000°C for all three curves, the initial radii of which are 0.5, 0.1 and 0.01 mm. It is observed that the originally smaller droplets, which have larger total surface area, evaporate in a small fraction of the time required by the ones with bigger original sizes.

Computations were also made for evaporating mercury droplets. Liquid mercury has a density approximately 13.5 times that of water and its latent heat of vaporization is approximately 9 times smaller than that of water. The vapor conductivity and Prandtl number of the two substances differ by smaller amounts, although mercury vapor is more viscous than water vapor. Figure 6 shows the evaporation of mercury droplets of different initial radii (1, 0.1 and 0.01 mm) for a volume concentration of 0.00419. Again it is observed that smaller droplets evaporate much faster than larger ones, because of the larger total surface area.

Comparisons of the results for the water and mercury droplets were made. In order to make the comparisons as meaningful as possible the following initial parameters were the same in both cases: (a) the number of droplets N, (b) the initial radius r_0 (the above two conditions imply the same initial concentration C_0 also), (c) the initial loading m_0^*, (d) the initial Reynolds number Re_0 and (e) the temperature step $(T_w - T_{g0})$. Thus, the differences in the results observed are due to the differences in the properties of the two substances and not on the choice of the initial conditions.

Figure 7 shows the evaporation of the droplets of the two substances for initial radii of 0.1 mm, $N = 10^9$ particles per cubic meter and temperature step of 580°C. It is observed that mercury evaporates faster than water, although the difference in the total time of evaporation is approximately 10%.

Figure 8 shows the evolution of the dimensionless gas velocities for the two substances up to the point when the droplets evaporate completely. The conditions for these calculations are the same as in the previous figure, the only difference being that the temperature step imposed is 380°C. It must be pointed out that at the end of the evaporation process the dimensionless gas temperature (T_g^*) for the mercury case is 0.83, while that of the water case is 0.74. Therefore, it is expected that when equilibrium is reached finally in both cases, the dimensionless velocity in the water will be about 30% higher than that of mercury.

The dimensionless gas temperature T_g^* is plotted in Fig. 9 for both the substances under the same con-

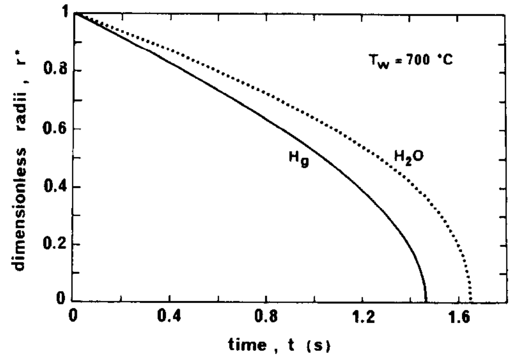

FIG. 7. Comparison of evaporation rates for water and mercury for a temperature step of 580°C and with $C_0 = 0.00419$ and $T_p = 120°C$.

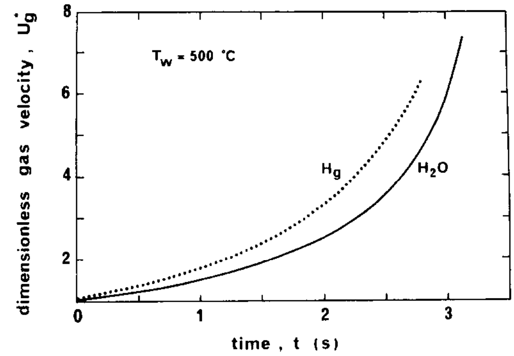

FIG. 8. Dimensionless velocities for water and mercury during evaporation for a temperature step of 380°C, with $C_0 = 0.00419$ and $T_p = 120°C$.

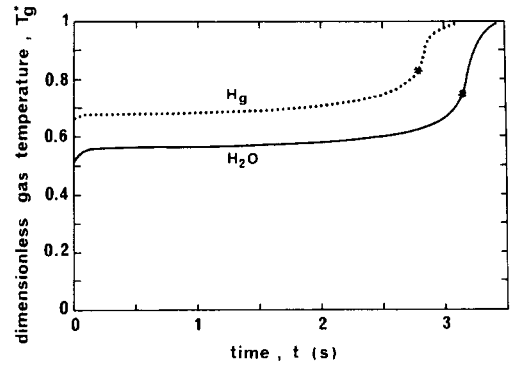

FIG. 9. Dimensionless gas temperatures for water and mercury for a temperature step of 380°C, with $C_0 = 0.00419$ and $T_p = 120°C$.

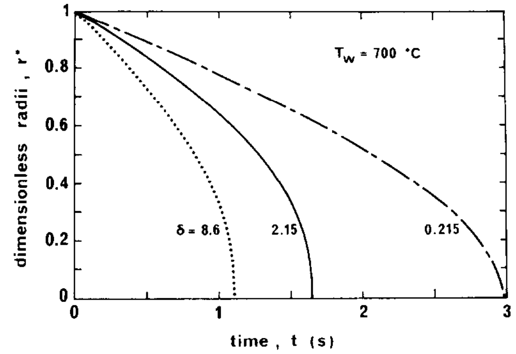

FIG. 10. The effect of the parameter δ on the rate of evaporation for a wall temperature of 700°C, with $C_0 = 0.00419$ and $T_p = 120°C$.

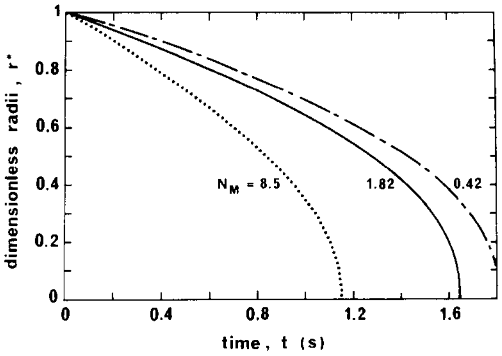

FIG. 11. The effect of the parameter N_M on the rate of evaporation for a wall temperature of 700°C, with $C_0 = 0.00419$ and $T_p = 120°C$.

ditions as in Fig. 8. The points with stars denote the end of the evaporation process for the droplets. Above the starred points there is single-phase heat transfer.

One of the parameters which influences the heat transfer characteristics of a particulate mixture is the ratio of the specific heats δ, which is approximately 0.48 for water droplets. Figure 10 shows the influence of this parameter on the evaporation of droplets. The three curves shown are for $\delta = 8.6$, 2.15 and 0.215 with $T_w = 700°C$ and all the other initial parameters as those in Fig. 1. It is seen that a high δ increases the rate of evaporation of the droplets by a considerable amount.

The dimensionless group N_M takes into account some of the thermal properties of the substances in the equation for mass evaporation. The value of this group (for $T_w = 700°C$) for water is 1.82 and for mercury is 0.446. The influence of this dimensionless group alone is shown in Fig. 11, which is drawn for $N_M = 8.5$, 1.82 and 0.42. It is seen that a high N_M causes an increase in the rate of phase change of the particles or droplets, albeit by a small amount.

CONCLUSIONS

This paper shows the development of a mathematical model for the motion of particles or droplets in a duct, while they undergo a phase change. The model treats the two phases separately and allows for thermal and mechanical non-equilibrium. The conservation and closure equations are derived and transformed in their dimensionless form. The dimensionless groups that influence the phase change and heat transfer in the two phases include as usual the Reynolds, Prandtl and Nusselt numbers for the flow as well as three groups related to the properties of the two phases, δ, N_M and N'_M.

The solution of the system of differential equations was accomplished for two substances having the properties of water and mercury and for the case of a step in the wall temperature. The results showed that the rate of phase change increases with higher wall temperature or lower particle diameter as well as with higher δ and N_M. The temperature of the gaseous phase remains almost constant, during the time when phase change occurs and increases rapidly when the phase change is completed. Therefore, during most of the phase change process the temperatures of both phases remain almost constant (although the two are different). The velocities of the two phases increased considerably in all the cases because of the combined effect of evaporation and increase of the gaseous temperature.

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ECOULEMENT AVEC SUBLIMATION OU ÉVAPORATION DE PARTICULES EN SUSPENSION, DANS LE CAS DE NON-ÉQUILIBRE THERMIQUE ET HYDRODYNAMIQUE

Résumé—On développe un modèle qui permet de décrire la sublimation d'une particule solide ou l'évaporation d'une goutte, dans l'écoulement d'un gaz. On montre le système d'équations à utiliser pour décrire le phénomène; on écrit aussi ces équations dans leur forme non-dimensionnelle. Le système d'équations obtenu est résolu numériquement pour le cas où le mélange de deux phases est soumis à un saut de température de la paroi. On étudie alors d'effet de différents paramètres (comme la taille de particules, leurs propriétés physiques, l'importance du saut de température) sur la vitesse d'évaporation de la phase condensée. Les températures et les vitesses de deux phases durant le changement de phase sont calculées et représentées sur les figures.

PARTIKELSTRÖMUNG MIT SUBLIMATION ODER VERDAMPFUNG IN THERMISCHEM UND HYDRODYNAMISCHEM NICHTGLEICHGEWICHT

Zusammenfassung—Es wird ein Modell entwickelt für die Beschreibung der Sublimation an Partikeln oder der Verdampfung an Tropfen in einer gasförmigen Umgebung. Die Erhaltungs- und Schließ-Gleichungen werden entwickelt und in dimensionslose Form gebracht. Das entstehende Gleichungssystem wird numerisch gelöst, und zwar für den Fall, daß das zweiphasige Gemisch einer stufenweisen Erhöhung der Wandtemperatur ausgesetzt wird. Der Einfluß verschiedener Parameter (Partikelgröße, Stoffeigenschaften und Große der sprunghaften Änderung der Wandtemperatur) auf die Verdampfung der kondensierten Phase wird untersucht. Ebenso werden die Temperatur und die Geschwindigkeit der beiden Phasen während des Phasenwechsels berechnet.

СУБЛИМАЦИЯ И ИСПАРЕНИЕ ЧАСТИЦ В НЕРАВНОВЕСНЫХ УСЛОВИЯХ

Аннотация—Разработана модель для описания процесса сублимации частиц или испарения капель в газообразной среде. Выведены уравнения сохранения и замыкания в безразмерном виде. Полученная система уравнений решена численно для случая, когда смесь двух фаз подвержена действию скачкообразного роста температуры стенки. Исследовано влияние нескольких параметров (таких как размеры частиц, физические свойства и вели чина скачка температуры стенки) на интенсивность испарения конденсированной фазы. Рассчитаны и приведены температуры и скорости обеих фаз в процессе фазового перехода.